

The synoptic problem and statistics

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In New Testament studies, the gospels of Matthew, Mark and Luke are known as the *synoptic gospels*. Especially when their texts are laid out side by side in a suitable format, the synoptic gospels are seen to share much common material, although there are also many differences. The gospel of John, although it tells broadly the same overall story, is written in a very different style, and there is not such a close correspondence between its text and those of the synoptic gospels. We shall focus here on the synoptic problem, which is concerned with hypotheses that attempt to explain the relationships between the synoptic gospels.

The texts of the gospels may be partitioned into sections, referred to as *pericopes* by biblical scholars. Each such pericope is a reasonably self-contained section of text, which may be a section of narrative material or a section of teaching, such as a parable, or a combination of both. Naturally, there are some differences of opinion as to how the text should be partitioned, but on the whole there seems to be broad agreement about the specification of most of the pericopes.

Some of the pericopes are unique to just one of the synoptic gospels, and such material is known as *single tradition*. So, for example, the birth and infancy narratives of the first two chapters of Luke's gospel, including the familiar Christmas story of the birth of Jesus and of the appearance of the angel to the shepherds, are single tradition material. So too are the very different birth and infancy narratives of the first two chapters of Matthew, including the story of the visit of the wise men, the Magi. Mark, on the other hand, has no birth and infancy narrative at all.

Other pericopes are common to just two of the synoptic gospels, and they are known as *double tradition*. The details of the wording of such pericopes will nevertheless differ to a greater or lesser extent between the gospels. The pericopes that make up Matthew's famous Sermon on the Mount are predominantly double tradition, in that they are to be found also in the gospel of Luke but not in Mark. However, although they are gathered together in a single block of teaching in Matthew, they are scattered in different locations in Luke. The majority of double tradition pericopes are those that are common to Matthew and Luke, and sometimes the term *double tradition* is restricted to these, but there are smaller numbers of double tradition pericopes that are common to Mark and Matthew or to Mark and Luke. Finally, there is the *triple tradition* of pericopes that are common to all three synoptic gospels, which include a great variety of accounts of healings, miracles and the teaching of Jesus, and most of the passion narrative.

A standard tool in the comparative study of the gospels is the *synopsis*, a book in which the gospels are printed in parallel columns on the page, pericope by pericope, so that comparisons of the wording may readily be made. The synopsis nowadays most commonly used by biblical scholars is that of Aland (1996), which is based on the Greek text, but, for non-specialists, a readily available and clearly laid out synopsis is that of Throckmorton (1992), which is based on the NRSV English translation of the bible. The layout of synopses may vary considerably, as is the case when Aland and Throckmorton are compared, partly because of variant specifications of the pericopes but also because the order in which the pericopes appear varies from gospel to gospel. Because of this

latter fact, different compilers of synopses may choose different orderings of the pericopes in their presentation of the material.

Because of the complex patterns of similarities and dissimilarities between the synoptic gospels, the problem of how to account for the relationships between the gospels is a notoriously difficult one in New Testament studies. To what extent has any gospel writer used the gospels of his predecessors and what other sources, oral or written, may he have had? Little is known about the history of the early church in the second half of the first century, when the synoptic gospels were probably written, and the time and place of writing of any of the gospels is highly conjectural, although some indications are given by church traditions from later centuries. Because of this, hypotheses about the relationships between the synoptic gospels are based almost entirely upon the internal evidence of the texts themselves. On the other hand, any such hypothesis will have implications for our understanding of early church history. A helpful introduction to the issues involved and the various models that have been proposed is given by Goodacre (2001) and further information may be found at Stephen Carlson's synoptic problem website at www.hypotyposesis.org/synoptic-problem/.

In the modern era of critical biblical scholarship, the first hypothesis to gain a large degree of acceptance was the *Griesbach hypothesis*, which was the dominant one in the late eighteenth and early nineteenth century. According to Griesbach, Matthew's was the first gospel to be written. Matthew was used by Luke, and Mark was a conflation of Matthew and Luke. In the nineteenth century there emerged the *two-source hypothesis*, according to which Mark's was the first surviving gospel to be written. Mark was used independently by Matthew and Luke, but they also had another hypothetical source *Q*, which has not survived but which accounts for the large quantity of double tradition material common to Matthew and Luke but absent from Mark. The two-source hypothesis became the dominant one and remains so to the present day, so that textbooks often present it as more or less established fact. Indeed, there is a scholarly industry devoted to reconstructing the lost text of *Q* and even providing a historical and social setting for its development through a series of editions. However, over the last few decades, a serious challenge has been mounted to the two-source hypothesis, especially by a revival of the Griesbach hypothesis and by the emergence of what is known as the *Farrer theory*, according to which Mark was the first gospel to be written, Matthew used Mark, and Luke used both Mark and Matthew.

Turning now to specifically statistical aspects of the synoptic problem, a classic and still very useful handbook for students of the synoptic problem is Hawkins' *Horae Synopticae* (1899, 1909), whose very title ("Synoptic Hours") points to the innumerable hours that the author spent poring over the texts of the gospels. It contains a wealth of data about the synoptic gospels, including statistics of word frequencies to demonstrate what words and phrases are particularly characteristic of each evangelist. Hawkins was a long-standing member of the influential Oxford Seminar on the synoptic problem, and, looking at the title page, where the author's name is given in full as "Rev. Sir John C. Hawkins, Bart.", one is taken back to an age of scholars and gentlemen.

Some of the arguments about the relative merits of the various hypotheses about synoptic relationships have been based upon the differences in order of the pericopes in the three synoptic gospels. To use an illustration which may be helpful to those who have studied elementary combinatorial mathematics, we may think of the pericopes as beads, which have been strung together on a string, in a different order in each gospel. There is

potential here for more mathematical approaches to the characterization of the differences in order between the gospels and in the evaluation of the arguments from order that have been made for some of the synoptic hypotheses.

In Abakuks (2006) a different statistical problem was investigated. Honoré (1968) in a pioneering paper had carried out a wide-ranging statistical analysis of the synoptic problem. Like Hawkins, Honoré must have spent many hours working through his synopsis, counting verbal agreements between the gospels. (Incidentally, what makes this effort even more remarkable is that Tony Honoré is a lawyer and not a biblical scholar, and this paper of his represents a one-off foray into New Testament studies. Between 1971 and 1988 he was Regius Professor of Civil Law in the University of Oxford. Now well into his eighties, he continues to teach and write.) A verbal agreement refers to a common occurrence in the same context in two or all three gospels of the same Greek word in the same grammatical form. For the purposes of the present analysis, we shall aggregate such verbal agreements over the union of the triple tradition and double tradition, that is, the whole of the synoptic material less the single tradition. This set of data includes all the material where there appear to be some links between the synoptic gospels, but excludes blocks of material which are unique to any gospel author. Table 1 gives the counts of words classified according to their presence or absence in each of the synoptic gospels for the triple and double tradition combined. In any row of Table 1, the count refers to the number of words that are present in the gospels marked with the number 1 but absent in the gospels marked with the number 0. Standard abbreviations, Matthew = Mt, Mark = Mk, Luke = Lk, are used.

Table 1. Counts of words in the triple and double tradition combined.

Mt	Mk	Lk	count
1	1	1	1852
1	1	0	2735
1	0	1	2386
0	1	1	1165
0	0	1	7231
0	1	0	5269
1	0	0	7588

One part of Honoré’s paper dealt with an innovative analysis of the so-called triple-link model. In what follows, like Honoré, we use the terms Gospel A, B and C to refer to any permutation of the synoptic gospels. It is supposed in the triple-link model that Gospels B and C both use Gospel A and that Gospel C also uses Gospel B. Let x be the probability that a given word in A is transmitted unaltered to B. Let y be the probability that a given word in B is transmitted unaltered to C. Let z be the probability that a given word in A is transmitted unaltered directly to C. The relationship is illustrated in Fig. 1 below.

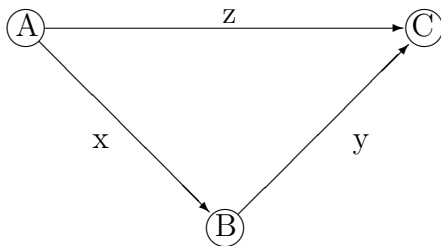


Fig. 1. The triple-link model

For example, the identification $A = \text{Mk}$, $B = \text{Mt}$, $C = \text{Lk}$ corresponds to the Farrer theory and the identification $A = \text{Mt}$, $B = \text{Lk}$, $C = \text{Mk}$ to the Griesbach hypothesis. Of the other possibilities, the most familiar one is the so-called Augustinian hypothesis that corresponds to $A = \text{Mt}$, $B = \text{Mk}$, $C = \text{Lk}$. However, the commonly accepted two-source hypothesis is not accommodated within the framework of the triple-link model.

Honoré made some further assumptions and then proceeded to carry out a mathematical and statistical analysis to fit his model to the data. In this he made some progress but ultimately went astray, essentially because of his lack of a sufficiently well-defined specification of the model in mathematical terms. In Abakuks (2006), Honoré’s assumptions and analysis were recast in terms of the notation of probability theory. Denote by A , B and C the events that a given word is in Gospel A, Gospel B and Gospel C, respectively. Further, denote by C_1 the event that the given word is in Gospel C and has been transmitted via Gospel B and denote by C_2 the event that the given word is in Gospel C and has been transmitted directly from Gospel A.

With this notation, $\Pr(B|A)$ denotes the conditional probability that a given word is in Gospel B given that it is in Gospel A. Using the basic definition of conditional probability,

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)},$$

this conditional probability may be evaluated directly from the data by the corresponding relative frequency, that is, the ratio of the number of words that are in both Gospels A and B to the number of words that are in Gospel A. The conditional probability so evaluated is precisely the probability that, for the aggregated triple and double tradition material, a word chosen at random from Gospel A is also in Gospel B — in the same context and in the same grammatical form. Similar direct evaluations can be made for all conditional probabilities involving A , B and C , but conditional probabilities that involve C_1 and C_2 have to be evaluated indirectly.

In terms of the notation that we have introduced, the probabilities x , y and z may be expressed as

$$\begin{aligned} x &= \Pr(B|A), \\ y &= \Pr(C_1|B), \\ z &= \Pr(C_2|A). \end{aligned}$$

It is straightforward to evaluate x directly, but expressions that may be used to evaluate y and z need to be derived using Honoré further assumptions, which in our terms amount to the following three conditional independence assumptions.

Assumption 1 – given that a word is in Gospel A, the event that it is transmitted to Gospel B and the event that it is transmitted directly from Gospel A to Gospel C are independent.

Assumption 2 – given that a word is in Gospel B, the event that it is in Gospel A and the event that it is transmitted from Gospel B to Gospel C are independent.

Assumption 3 – given that a word is in Gospel A and Gospel B, the event that it is transmitted from Gospel B to Gospel C and the event that it is transmitted directly from Gospel A to Gospel C are independent.

Furthermore, given these assumptions, we can find formulae for the probabilities that if a given word is in Gospel A then it is also in Gospels B and C and that if it is Gospel A then it is also in Gospel C: $\Pr(B \cap C|A) = xy + xz - xyz$ and $\Pr(C|A) = z + xy - xyz$. These values can be evaluated and compared with the values of $\Pr(B \cap C|A)$ and $\Pr(C|A)$ as calculated directly from the data. Following the approach of Honoré, the ratios of the values as calculated from the formulae to the values calculated directly may be used as a measure of the goodness of fit for each of the six possible variants of the model. The closer these ratios are to one, the better the fit of the model. The results are presented in Table 2.

Table 2. Evaluation of the triple-link model

A-B-C	x	y	z	$\Pr(B \cap C A)$			$\Pr(C A)$		
				$xy + xz - xyz$	direct	ratio	$z + xy - xyz$	direct	ratio
Mt-Mk-Lk	0.315	0.193	0.239	0.122	0.127	0.957	0.286	0.291	0.981
Lk-Mk-Mt	0.239	0.374	0.248	0.126	0.147	0.862	0.315	0.335	0.940
Mk-Mt-Lk	0.416	0.248	0.181	0.160	0.168	0.952	0.266	0.274	0.970
Lk-Mt-Mk	0.335	0.286	0.139	0.129	0.147	0.882	0.221	0.239	0.927
Mt-Lk-Mk	0.291	0.165	0.265	0.112	0.127	0.883	0.300	0.315	0.953
Mk-Lk-Mt	0.274	0.276	0.342	0.143	0.168	0.853	0.392	0.416	0.941

It appears that the Mt-Mk-Lk model, which corresponds to the Augustinian hypothesis, and the Mk-Mt-Lk model, which corresponds to the Farrer theory, give the best fit. These models also satisfy the criterion $x > \max(y, z)$, which, although it is not necessary to adopt, does seem a plausible one, since we might expect B to make more use of A than C to make use of each of the two sources A and B that he has at his disposal.

Although already in the second century the Christian scriptures came to be written in *codex*, i.e., book form, the gospels would originally have been written on scrolls, which were expensive and hard to come by and also awkward to handle. Partly from consideration of the physical conditions under which the gospels would have been written, in later work I have suggested a modification of Honoré’s model in which the Assumption 3 of conditional independence is replaced by an Assumption 3A of mutual exclusion:

Assumption 3A – the event that a word is transmitted from Gospel B to Gospel C and the event that it is transmitted directly from Gospel A to Gospel C are mutually exclusive.

This leads to simpler expressions for the evaluation of y and z and also to the simpler formulae $\Pr(B \cap C|A) = x(y + z)$ and $\Pr(C|A) = xy + z$. The results for the modified model are presented in Table 3.

Table 3. Evaluation of the modified triple-link model

A-B-C	x	y	z	Pr($B \cap C A$)			Pr($C A$)		
				$x(y+z)$	direct	ratio	$xy+z$	direct	ratio
Mt-Mk-Lk	0.315	0.174	0.239	0.130	0.127	1.024	0.294	0.291	1.010
Lk-Mk-Mt	0.239	0.348	0.248	0.142	0.147	0.972	0.331	0.335	0.988
Mk-Mt-Lk	0.416	0.234	0.181	0.173	0.168	1.028	0.278	0.274	1.017
Lk-Mt-Mk	0.335	0.275	0.139	0.139	0.147	0.946	0.231	0.239	0.967
Mt-Lk-Mk	0.291	0.150	0.265	0.121	0.127	0.949	0.309	0.315	0.980
Mk-Lk-Mt	0.274	0.254	0.342	0.163	0.168	0.970	0.411	0.416	0.988

Overall, the modified model appears to fit better, and if additionally the criterion $x > \max(y, z)$ is imposed then the Mt-Mk-Lk and Mk-Mt-Lk models again give the best fit.

The triple-link model, whether in its original or in its modified form, represents, of course, a radical simplification of the actual process of gospel composition, but it does provide a basis for the analysis of the data at our disposal here, and the results may provide some useful input to the discussion of the relative merits of the synoptic hypotheses that correspond to the variants of the triple-link model.

We have adopted an approach in which the word-count data are aggregated over the whole of the triple plus double tradition material, but further progress might be made at a more detailed level where the statistical data from individual pericopes are examined to investigate the plausibility of the various synoptic hypotheses.

References

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