Can Quality Ladders Explain Observed Engel Curves?

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Abstract

Substitution to higher-quality goods as wealth increases implies that virtually every single consumption good becomes inferior at some level of wealth. This paper generalizes this process to explain the observed shape of any Engel curve, including those relative to large aggregates of commodities. The idea is that consumption expenditure shares are influenced by the quality-upgrading process. A change in wealth, producing (possibly) different variations in quality levels across distinct goods, causes heterogeneous responses in those goods' expenditure shares. The shape of the Engel curve bends upwards or downwards depending on whether the quality rise in the relevant good is higher than the average qualitative increase of the whole consumption bundle. As a result, some goods may be considered as luxuries at any level of wealth while others may be considered so only for some wealth intervals, depending of the relative cost structure of quality-upgrading across the different goods. The proposed theory may open new debates in the literature of the economics of innovation, and in those fields where recent contributions show the importance of non-homothetic demand, namely economic growth and international trade.

1 Introduction

Of all the empirical regularities observed in economic data, Engel's Law is probably the best established. (H.S. Houthakker)

It is a fact – as proven by substantial empirical evidence since Engel's (1895) Law – that consumption expenditure shares are wealth-varying. In a couple of recent contributions, Matsuyama (2000, 2002) points out the crucial role that wealth effects on demand play in economic growth and international trade issues. Surprisingly, economic theory is rather vague in explaining a phenomenon of this importance. In particular, it appears to neglect exploring the determinants of observed Engel curves.[1] This paper proposes a theory for wealth effects on demand based

[1] An Engel curve is a function that describes how demand for a good relates to individuals' wealth.

on a process of substitution to higher-quality goods as wealth increases. The intuition is that quality attracts demand, hence spending is set in such a way that demand is stronger for those categories of goods (or *varieties*) whose quality levels are higher. As wealth increases, variations in the distribution of spending follow from the fact that substitution to higher-quality goods occurs at different rates across varieties.

The idea of linking expenditure shares variations to the quality upgrading process originates from observing that Engel curves for single commodities eventually bend downwards.[2] As Mas-Colell, Whinston, and Green (1995) point out, the "assumption of normal demand makes sense if commodities are large aggregates (e.g., food, shelter). But if they are very disaggregated (e.g., particular kinds of shoes), then because of substitution to higher-quality goods as wealth increases, goods that become inferior at some level of wealth may be the rule rather than the exception." It follows that single commodities may be considered as normal or inferior goods depending on the level of individuals' wealth, and this argument automatically extends to luxuries and necessities.[3] Under this perspective, the quality-upgrading process is responsible for variations in the shape of Engel curves, at least at a one-commodity level. Our approach generalises this argument to varieties, which observed Engel curves typically refer to.

The fact that varieties are not inherently inferior, necessary or luxury is largely supported by empirical evidence. Although early analyses, e.g. Working (1943) and Leser (1963), support a linear specification of Engel curves, more recent studies, e.g. Lewbel (1991), Hausman, Newey and Powell (1995) and Banks, Blundell and Lewbel (1997), document nonlinearity in Engel curves, including quadratic or S-shapes. These findings suggest that varieties may change "status", switching from normal to inferior, or from luxury to necessary, or vice versa. This paper proposes a theoretical rationale for such "status" variations, building a bridge between quality ladder models – from which the quality-upgrading process is drawn, but which usually deliver homothetic demand functions, see e.g. Aghion and Howitt (1992), and Grossman and Helpman (1991b) – and frameworks that obtain non-homotheticity from hierarchic specifications of preferences.[4]

[2] Here, Engel curves are intended in their *total expenditure* form. A downward sloping Engel curve represents demand for an inferior good, as opposed to a normal good, which is characterised by an upward sloping Engel curve. Recall that inferior goods are defined as those commodities whose demand decreases with rising wealth. Conversely, demand for normal goods increases with rising wealth.

[3] Among the normal goods, those whose demand increases *more than proportionally* with rising wealth are referred to as luxuries; the others as necessities. In short, "goods with income elasticities of demand below zero, between zero and one, and above one are called inferior goods, necessities, and luxuries respectively." (Lewbel, 2006, p.2)

[4] First formalized by Jackson (1984) on the basis of the tree-structured utility functions proposed by Strotz (1957), hierarchic preferences specification has been recently applied to the analysis of several macroeconomic issues, such as the role of wealth distribution in the adoption of new technologies (Murphy, Shleifer, and Vishny, 1989; Zweimuller, 2000), in the determination of economic growth performance (Falkinger, 1994; Foellmi and Zweimüller, 2006), and in the formation of the international trade structure (Mitra and Trindade, 2005). Matsuyama (2000, 2002) also adopts a hiararchic specification. Other contributions where preferences have a nonhomothetic structure include Aoki and Yoshikawa (2002), who investigate economic growth issues construct-

The rest of the paper is organized as follows. The next section presents some evidence on spending distribution in the U.S. for different wealth levels and for selected categories of goods. Section 3 describes the commodity set, and illustrates the structure of the model. Section 4 characterizes the equilibrium. Section 5 illustrates the wealth effects on demand. Section 6 concludes. The Appendix contains a definition of quality based on Lancaster's (1966) characteristics theory.

2 Observations on Demand Composition

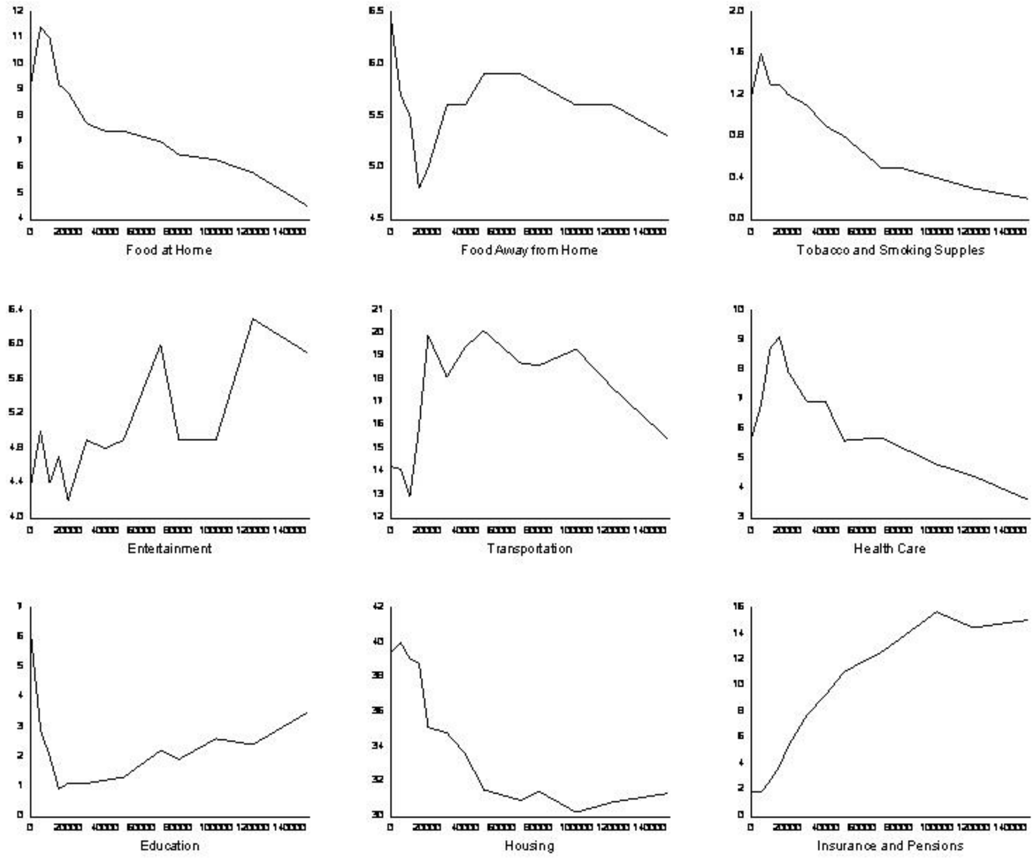
This section illustrates how expenditure shares for nine categories of goods vary as wealth increases in the U.S. final market. Accordingly, Engel curves are represented in their *expenditure share* form. In this form, a downward sloping Engel curve may refer either to a necessity or to an inferior good – depending on the function's curvature, at an aggregate level such as used here, however, inferiority is easily neglected – whereas an upward sloping Engel curve refer to a luxury. Recall that an expenditure share measures the percentage of wealth devoted to consumption of a given variety. If this percentage increases with rising income, then the spending on that category of goods increases more than proportionally with wealth, implying that the relevant income elasticity of demand is greater than one. As a result, that variety should be considered as a luxury. If this is to be considered as an inherent property of the category of goods, then the expenditure share should increase monotonically with rising income. Naturally, the opposite argument applies for necessities.

The fact that such a monotonicity does not consistently hold in observed data can be inferred simply by considering a few cross-sectional examples on the evolution of the expenditure shares as wealth increases. Figure 1 portrays these paths for selected categories of goods in 2005. The x-axis measures wealth, whereas the y-axis reports the expenditure shares. Data are provided by the U.S. Bureau of Labor Statistics. The portrayed categories are chosen as to reflect the most interesting ones for the purposes of this paper. By examining Figure 1, it is easy to infer that variations in the income proportions are not monotonic.

Most of the trends are confirmed, at least for some wealth ranges. For instance, expenditure shares for food at home, tobacco and smoking supplies, and health care generally decline with rising wealth; the income proportion spent on entertainment increases, along with those devoted to education and to insurance and pensions. Accordingly, income elasticities are greater than one for the first group of categories, and smaller than one for the second group. Hence, the former should be considered as necessities; the latter as luxuries.

ing a model based on logistic Engel curves; Galor and Moav (2004), who analyse household's bequest behaviour; and Voigtländer and Voth (2005), who apply nonhomothetic preferences to a historical context.

FIGURE 1 – Income Proportion Variations as Wealth Increases



The examination of Figure 1 suggests, however, that luxuriousness and necessity cannot be considered as inherent properties of the different varieties. Virtually all categories appear to change "status" at least once as wealth increases. For example, expenditure shares for food at home, tobacco and smoking supplies, and health care increase at the lowest levels of wealth, declining monotonically only after reaching a certain wealth threshold; the opposite occurs for education. In addition, the series display significant fluctuations: in particular, expenditure shares for food away from home and transportation first decline, then rise, then decline again as wealth increase; that for entertainment oscillates twice as much. Accordingly, income elasticities are not steadily greater or smaller than one, thus varieties appear to become luxuries (from necessities) of necessities (from luxuries) at some level of wealth– that is, luxuriousness and necessity do not seem inherent properties of commodities, not even at an aggregate level.

3 The Model

The economy can produce a potentially infinite number of goods. Following Grossman and Helpman (1991), the goods are organised in a commodity space defined along two distinct dimensions: *horizontal* and *vertical.* The first dimension (*horizontal*) indexes *varieties*, which are categories of goods denoted by the letter v along the variety space $\mathbb{V} \subset \mathbb{R} : v \in [0,1]$. The second dimension (*vertical*) orders goods, within each variety $v \in \mathbb{V}$, according to their intrinsic *quality*, denoted by q and belonging to the set $\mathbb{Q} \subseteq \mathbb{R}_{+}$. The commodity space is then given by the set $\mathbb{V} \times \mathbb{Q} = [0,1] \times [0,\infty)$, and each commodity is identified by a pair $(v,q) \in \mathbb{V} \times \mathbb{Q}$.[5]

The demand side of the economy is inhabited by a continuum of individuals with identical preferences defined over the commodity space $\mathbb{V} \times \mathbb{Q}$. Individuals are endowed with a given amount of resources, denoted by w and expressed in terms of a *numeraire*. Preferences are represented by the utility function:

$$U = \int_{\mathbb{V}} q_v \ln x_v dv, \tag{1}$$

where $q_v \in \mathbb{Q}$ represents the consumed quality of variety v, and $x_v \in \mathbb{R}_{+}$ is the *quantity* of consumption for quality q_v of variety v.[6] The assumption of a single commodity consumed for each variety, which is standard in the quality ladder literature, is taken to simplify the modelling of the process of substitution to higher-quality goods as wealth increases, and corresponds to considering commodities of one variety as perfect substitutes.[7] Notice that utility (1) is obtained as a particular case of the more general isoelastic utility function of Dixit and Stiglitz (1977) type:

$$U = \frac{1}{1-\sigma} \left\{ \left[\int_{\mathbb{V}} q_v \left(x_v\right)^a dv \right]^{\frac{1-\sigma}{\alpha}} - 1 \right\},$$

once it is assumed that marginal utility of one variety is independent of consumption of other varieties ($\alpha = 1 - \sigma$), and that the curvature of utility equals one ($\sigma = 1$).[8]

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[5] In our setup, commodities in different varieties should be then understood as goods exhibiting a limited degree of substitutability among each other. On the other hand, different qualities in a particular variety should be regarded as close substitutes.

[6] The quantity of consumption for all other qualities is set to zero, i.e. $x_{v,q} = 0$, $\forall q \neq q_v$. Notice that, to be more precise, preferences should be represented by the utility function:

$$U = \int_{\mathbb{V}} \ln\left[\max\left\{x_v, \left(x_v\right)^{q_v}\right\}\right] dv$$

to avoid quality being regarded as a *bad* when (physical) consumption is sufficiently low ($x_v < 1$). As shown in Jaimovich and Merella (2007), however, no difference in the equilibrium conditions arises by replacing (1) with this function, as long as low quality goods are sufficiently cheaper.

[7] Alternatively, quality q_v and quantity x_v can be respectively thought of as average quality and total physical quantity of consumption for a bundle of commodities of the generic variety v.

[8] Another assumption is actually required, namely $\int_{\mathbb{V}} q_v dv = 1$. This assumption can be, however, avoided by using the ordinal property of utility, according to which U and U/Q, with $Q = \int_{\mathbb{V}} q_v dv > 0$, represent the same

Individuals' budget constraint reads:

$$\int_{v\in\mathbb{V}} p_v x_v dv \leq w, \tag{2}$$

where p_v is the price of commodity q_v.

The supply side of the economy is populated by infinite identical firms, each potentially producing one of the commodities in the set $\mathbb{V}\times\mathbb{Q}$. Firms' production consists of transforming *raw materials* into different final goods, according to commodity-specific technologies. Firms extract raw materials at a fixed marginal cost, which is assumed to equal one, so that raw materials can be taken as the *numeraire* of the economy. The production technology for each commodity is assumed to be linear in the amount of raw materials employed, so that the marginal cost is a function of degree zero in the quantity produced. Free entry in all markets assures that the price of each actively consumed commodity, denoted by p_v for commodity q_v, is kept at the level of marginal cost, generically denoted by $c(v, q_v)$, that is:[9]

$$p_v = c(v, q_v) \tag{3}$$

Assumption 1 (Marginal Cost) *The function $c(v, q_v)$ is such that, for all $v \in \mathbb{V}$: 1) $c(v, 0) = 0$; 2) $\partial c(\cdot, q_v)/\partial v > 0$; 3) $\partial c(v, \cdot)/\partial q_v > 0$; 4) $\eta_v \equiv [\partial c(v, \cdot)/\partial q_v]\, q_v/p_v > 1$; 5) $\partial \eta_v/\partial q_v = 0$. In addition, 6) $\partial \eta_v/\partial v > 0$ must also hold.*

In short, Assumption 1 implies that varieties are ordered by rising cost of quality upgrading, and that the latter increases more than proportionally with rising quality (in turn implying that upgrades become increasingly costly as quality moves up the ladder.) In addition, the simplifying assumption of constant cost elasticity of quality upgrading is also taken.

4 Equilibrium and Wealth Effects on Demand

Given utility (1), the budget constraint (2) will naturally bind in equilibrium. Denoting the expenditure share for variety v by $\beta_v \equiv p_v x_v/w$, the relevant quantity can be rewritten as $x_v = \beta_v w/p_v$, and the budget constraint as $\int_{\mathbb{V}} \beta_v dv = 1$. The individuals' problem can be accordingly restated in terms of choosing optimal qualities $\{q_v^*\}_{v\in\mathbb{V}}$ and optimal expenditure shares $\{\beta_v^*\}_{v\in\mathbb{V}}$ rather than optimal quantities $\{x_v^*\}_{v\in\mathbb{V}}$.

preferences. In this case, we could write:

$$U = \frac{1}{1-\sigma}\left\{\left[\int_{\mathbb{V}} \frac{q_v}{Q}(x_v)^a dv\right]^{\frac{1-\sigma}{\alpha}} - 1\right\},$$

where $\int_{\mathbb{V}} (q_v/Q)\, dv = 1$ is satisfied by definition.

[9] Formally, denoting the amount of raw materials employed by m_v, the production technology for commodity q_v may be expressed by $m_v/c(v, q_v)$.

Problem 1 *The representative agent solves the following optimisation problem:*

$$\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad U = \int_{\mathbb{V}} q_v \ln\left(\frac{\beta_v w}{c(v,q_v)}\right) dv;$$

$$\text{subject to:} \quad \int_{\mathbb{V}} \beta_v dv = 1.$$

Solution. The Lagrangian for Problem 1 reads as follows:

$$\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}},\mu} \mathcal{L} = \int_{\mathbb{V}} q_v \ln\left(\frac{\beta_v w}{c(v,q_v)}\right) dv + \mu\left(1 - \int_{\mathbb{V}} \beta_v dv\right);$$

which yields to the following first-order-conditions:

$$\frac{\partial \mathcal{L}}{\partial q_v} = \ln\left(\frac{\beta_v^* w}{c(v,q_v^*)}\right) - \frac{q_v^*}{c(v,q_v^*)}\frac{\partial c(v,\cdot)}{\partial q_v} = 0, \ \forall v \in \mathbb{V}; \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial \beta_v} = \frac{q_v^*}{\beta_v^*} - \mu = 0, \ \forall v \in \mathbb{V}; \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = 1 - \int_0^1 \beta_v^* dv = 0.$$

the three equations above, taken simultaneously, represent the problem's solution. ■

The first order condition (4) solves, for each variety v, the trade-off implied by the choice of the optimal quality: the negative effect on utility generated through x_v by the rise in price p_v must be perfectly offset by the positive direct effect of quality. By considering the first order condition (5) for two generic varieties $v, z \in \mathbb{V}$, it is easy to obtain that the ratio of the two varieties' expenditure shares must equal the ratio of the two varieties' optimal qualities, i.e. $\beta_v^*/\beta_z^* = q_v^*/q_z^*$. A more general relationship between each expenditure share and optimal qualities can be found by denoting the aggregate index measuring the optimal consumption bundle's *average quality* by $Q \equiv \int_{\mathbb{V}} q_z^* dz$.

Lemma 1 (Optimal Expenditure Share) *The expenditure share devoted to consumption of a generic variety $v \in \mathbb{V}$ is proportional to the optimal quality of variety v. Formally:*

$$\beta_v^* = \frac{q_v^*}{Q}, \ \forall v \in \mathbb{V}. \tag{6}$$

Proof. Computing (**??**) for two generic varieties $v, z \in \mathbb{V}$, it follows that $\beta_v^* q_z^* = \beta_z^* q_v^*$. Integrating both sides across varieties yields $\beta_v^* \int_{\mathbb{V}} q_z^* dz = q_v^* \int_{\mathbb{V}} \beta_z^* dz = q_v^*$. Rearranging, and using the definition of average quality: $Q \equiv \int_{\mathbb{V}} q_z^* dz$, (6) obtains. ■

It follows that if all varieties were optimally consumed in the same quality (i.e. if $q_v^* = \bar{q}$, $\forall v \in \mathbb{V}$), then $\beta_v^* = 1$ would hold for all v. Lemma 1 points out that the optimal expenditure shares do not depend on some preordained preference ordering; they are rather set according to the distribution of the optimal qualities across varieties. The higher the quality of a given variety, the larger the proportion of wealth spent on that variety.

Using (6) into (4), an expression can be found that implicitly determines the value of the optimal quality for each variety. Problem 1 does not allow for closed form solutions for optimal qualities. It is, however, possible to characterise the distribution of quality across varieties.

Lemma 2 (Optimal Quality Distribution) *The optimal quality $q_v^* = q(v, w)$ of a generic variety $v \in \mathbb{V}$ is a decreasing function of the variety index. Formally:*

$$\frac{\partial q(v, w)}{\partial v} = -\frac{q_v^*}{\eta_v - 1}\left(\frac{\partial \eta_v}{\partial v} + \frac{1}{c(v, q_v^*)}\frac{\partial c(\cdot, q_v^*)}{\partial v}\right) < 0. \tag{7}$$

Proof. Considering (6), and recalling that $\eta_v \equiv [\partial c(v, \cdot)/\partial q_v]\, q_v/c(v, q_v)$, first order condition (4) can be rewritten as:

$$\eta_v + \ln c(v, q_v^*) - \ln q_v^* = \ln(w/Q), \ \forall v \in \mathbb{V}. \tag{8}$$

Total differentiating with respect to v yields:

$$\frac{\partial \eta_v}{\partial v} + \frac{1}{c(v, q_v^*)}\frac{\partial c(\cdot, q_v^*)}{\partial v} + \frac{1}{c(v, q_v^*)}\frac{\partial c(v, \cdot)}{\partial q_v}\frac{\partial q(\cdot, w)}{\partial v} - \frac{1}{q(v, w)}\frac{\partial q(\cdot, w)}{\partial v} = 0.$$

Isolating $\partial q(\cdot, w)/\partial v$, (7) obtains. The sign of $\partial q(\cdot, w)/\partial v$ is negative since, by Assumption 1, $\partial c(\cdot, q_v^*)/\partial v > 0$, $\eta_v > 1$ and $\partial \eta_v/\partial v > 0$. ■

Lemma 2 implies that quality is higher for those varieties whose quality upgrading process is cheaper (see Assumption 1). In addition, recalling Lemma 1, it is straightforward to notice that the distribution of expenditure shares mirrors that of qualities. As a result, expenditure shares are also higher for those varieties with a less costly substitution to higher-quality goods.

Along with the characterisation of the distribution of quality across varieties, Lemma 2 also points out, by denoting the optimal quality by $q_v^* = q(v, w)$ in accordance with (8), that the solution of Problem 1 fully accounts for an endogenous process of substitution to higher quality goods as wealth increases. The characterisation of the response of optimal qualities to changes in wealth can be obtained by differentiating (8) with respect to w.

Proposition 1 (Wealth Effect on Optimal Quality) *The optimal quality $q_v^* = q(v, w)$ of a generic variety $v \in \mathbb{V}$ is an increasing function of wealth. Formally:*

$$\frac{\partial q(v, \cdot)}{\partial w} = \frac{q_v^*/(\eta_v - 1)}{\int_{\mathbb{V}} \eta_z q_z^*/(\eta_z - 1)\, dz}\frac{Q}{w} > 0. \tag{9}$$

Proof. Differentiating (8) with respect to w yields:

$$\frac{1}{c\left(v,q_v^*\right)}\frac{c\left(\cdot,q_v\right)}{\partial q_v}\frac{\partial q\left(v,w\right)}{\partial w}-\frac{1}{q_v^*}\frac{\partial q\left(v,\cdot\right)}{\partial w}=\frac{1}{w}-\frac{1}{Q}\int_{\mathbb{V}}\frac{\partial q\left(z,\cdot\right)}{\partial w}dz.$$

Isolating $\partial q\left(v,w\right)/\partial w$, factoring, recalling that $\eta_v\equiv\left[\partial c\left(v,\cdot\right)/\partial q_v\right]q_v/c\left(v,q_v\right)$, and denoting the wealth effect on average quality by $\partial Q\left(\cdot\right)/\partial w\equiv\int_{\mathbb{V}}\partial q\left(z,\cdot\right)/\partial w dz$:

$$\frac{\partial q\left(v,\cdot\right)}{\partial w}=\left(\frac{1}{w}-\frac{1}{Q}\frac{\partial Q\left(\cdot\right)}{\partial w}\right)\frac{q_v^*}{\eta_v-1}. \tag{10}$$

Integrating across varieties, isolating $\partial Q\left(\cdot\right)/\partial w$, and rearranging:

$$\frac{\partial Q\left(\cdot\right)}{\partial w}=\frac{\int_{\mathbb{V}}q_z^*/\left(\eta_z-1\right)dz}{\int_{\mathbb{V}}\eta_z q_z^*/\left(\eta_z-1\right)dz}\frac{Q}{w}>0. \tag{11}$$

Finally, using (11) into (10), and rearranging, (9) obtains. The sign of $\partial q\left(\cdot,w\right)/\partial v$ is positive since $\eta_v>1$ and $q_v^*,Q,w>0$. ■

Proposition 1 implies that rising wealth has always a positive effect on the values of optimal qualities. In addition, since cost elasticities of quality upgrading and quality levels vary across varieties, so may wealth effects on optimal qualities do. The (possibly different) rates at which the substitution occurs across varieties can be characterised as follows.

Corollary 1 (Distribution of Wealth Effects on Quality) *The wealth effect on optimal quality $\partial q\left(v,\cdot\right)/\partial w$ of a generic variety $v\in\mathbb{V}$ is a decreasing function of the variety index. Formally:*

$$\frac{\partial^2 q\left(\cdot,\cdot\right)}{\partial v\partial w}=\left(\frac{1}{q_v^*}\frac{\partial q\left(\cdot,w\right)}{\partial v}-\frac{1}{\eta_v-1}\frac{\partial\eta_v}{\partial v}\right)\frac{\partial q\left(v,\cdot\right)}{\partial w}<0. \tag{12}$$

Proof. Differentiating (9) with respect to v yields:

$$\frac{\partial^2 q\left(\cdot,\cdot\right)}{\partial v\partial w}=\frac{1}{\int_{\mathbb{V}}\eta_z q_z^*/\left(\eta_z-1\right)dz}\frac{Q}{w}\left(\frac{q_v^*}{\eta_v-1}\frac{\partial q\left(\cdot,w\right)}{\partial v}-\frac{q_v^*}{\left(\eta_v-1\right)^2}\frac{\partial\eta_v}{\partial v}\right).$$

Rearranging, and using (9), (12) obtains. The sign of $\partial^2 q\left(\cdot,\cdot\right)/\partial v$ is negative since $\partial q\left(\cdot,w\right)/\partial v<0$, $\eta_v>1$ and $q_v^*,\partial\eta_v/\partial v,\partial q\left(v,\cdot\right)/\partial w>0$. ■

According to Corollary 1, not only are qualities levels higher for those varieties whose quality upgrading process is cheaper, but they also enjoy stronger wealth effects on optimal qualities. This result naturally implies that optimal qualities across varieties diverge with rising wealth.

From the joint consideration of Lemma 1 and Proposition 1, it straightforwardly follows that wealth variations affect the formation of expenditure shares devoted to the different varieties. Since expenditure shares must always sum up to one, however, this effect cannot share the same sign for all varieties. Defining $\tilde{\eta} = \left\{\eta_v : \int_{\mathbb{V}} q_z^* \left(\eta_z - \eta_v\right) / \left(\eta_z - 1\right) dz = 0\right\}$, and letting $\mathbb{L} = \{v \in \mathbb{V} : \eta_v < \tilde{\eta}\}$, the wealth effects on expenditures shares can be characterised as follows.

Proposition 2 (Wealth Effects on Expenditure Shares) *The optimal expenditure share $\beta_v^* = \beta(v, w)$ is a function of wealth, formally:*

$$\frac{\partial \beta(v, \cdot)}{\partial w} = \frac{\beta_v^*}{w \int_{\mathbb{V}} \eta_z q_z^* / \left(\eta_z - 1\right) dz} \int_{\mathbb{V}} \frac{\eta_z - \eta_v}{\eta_z - 1} q_z^* dz. \tag{13}$$

Such a function is increasing for each variety $v \in \mathbb{L}$; a non-increasing function otherwise.

Proof. Differentiating (6) with respect to w yields:

$$\frac{\partial \beta(v, \cdot)}{\partial w} = \frac{1}{Q} \frac{\partial q(v, \cdot)}{\partial w} - \frac{q_v^*}{Q^2} \frac{\partial Q(\cdot)}{\partial w}.$$

Using (9) and (11), and rearranging, (13) obtains. For any variety $v \in \mathbb{L}$, the sign of $\partial \beta(v, \cdot) / \partial w$ is positive because by definition $\eta_v < \tilde{\eta}$, hence $\int_{\mathbb{V}} q_z^* \left(\eta_z - \eta_v\right) / \left(\eta_z - 1\right) dz > \int_{\mathbb{V}} q_z^* \left(\eta_z - \tilde{\eta}\right) / \left(\eta_z - 1\right) dz$. The opposite argument holds for any variety $v \notin \mathbb{L}$. ■

Proposition 2 implies that rising income generally influences the optimal expenditure shares, though the sign of this effect is not univocal, and depends on the variety-specific cost of substitution to higher-quality goods relative to the benchmark $\tilde{\eta}$. Recalling Assumption 1, it is clear that the set $\mathbb{L}$ comprises the lower-indexed varieties. Hence, expenditure shares rise for those varieties whose quality upgrading process is cheaper, and fall for those whose substitution to higher-quality varieties is more costly.

Corollary 2 (Wealth Effects on $\tilde{\eta}$) *The cost elasticity of quality upgrading threshold $\tilde{\eta} = \eta(w)$ set of varieties is a decreasing function of wealth. Formally:*

$$\frac{\partial \eta(\cdot)}{\partial w} = \frac{\int_{\mathbb{V}} \frac{\eta_z - \tilde{\eta}}{\eta_z - 1} \frac{\partial q(z, \cdot)}{\partial w} dz}{\int_{\mathbb{V}} \frac{q_z^*}{\eta_z - 1} dz} < 0 \tag{14}$$

Proof. Recall that $\tilde{\eta} = \left\{\eta_v : \int_{\mathbb{V}} q_z^* \left(\eta_z - \eta_v\right) / \left(\eta_z - 1\right) dz = 0\right\}$. Differentiating with respect to w yields:

$$\int_{\mathbb{V}} \frac{\eta_z - \tilde{\eta}}{\eta_z - 1} \frac{\partial q(z, \cdot)}{\partial w} dz - \int_{\mathbb{V}} \frac{q_z^*}{\eta_z - 1} \frac{\partial \tilde{\eta}}{\partial w} dz = 0$$

isolating $\partial \tilde{\eta} / \partial w$, (14) obtains. The sign of $\partial \tilde{\eta} / \partial w$ is negative since, by Corollary 1, $\partial q(\cdot, \cdot) / \partial v \partial w < 0$, hence $\left| \int_{\mathbb{L}} \left[\partial q(z, \cdot) / \partial w\right] \left(\eta_z - \tilde{\eta}\right) / \left(\eta_z - 1\right) dz \right| > \int_{\not{\mathbb{L}}} \left[\partial q(z, \cdot) / \partial w\right] \left(\eta_z - \tilde{\eta}\right) / \left(\eta_z - 1\right) dz$ and $\int_{\mathbb{V}} \left[\partial q(z, \cdot) / \partial w\right] \left(\eta_z - \tilde{\eta}\right) / \left(\eta_z - 1\right) dz < 0$. ■

Corollary 2 implies that the set of varieties with positive wealth effects narrows as wealth increases. As a result, under the specification of quality-upgrading cost structure given by Assumption 1, there exists a subset of (higher-indexed) varieties which are always to be considered as necessities, and a complementary subset which is composed by varieties whose "status" switches from luxury to necessity as some level of wealth. The number of varieties to be considered as luxuries is in fact predicted to decrease with rising wealth.

The most important point is that the mechanism through which wealth variation affects the demand structure indicates that a commodity should be regarded as a luxury or as a necessity depending on supply-side factors, i.e. the cost of substitution to higher-quality goods as wealth increases, rather than on preference-based arguments, i.e. inherent properties of goods. As shown in the next section, along with a better fit of empirical evidence, this framework can explain changes in income elasticities without postulating variations in the structure of individuals' tastes.

5 A Numerical Example

COMING SOON

6 Concluding Remarks

COMING SOON

A Commodity Space and Definition of Quality

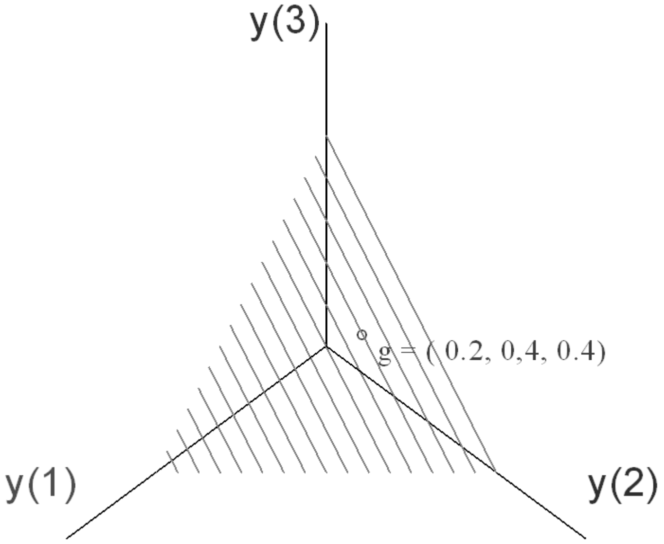
This section proposes a formal specification for the quality index. Alternative definitions can be found, among many others, in Stokey (1988) and Merella (2006). Although not essential – several contributions making explicit use of quality provide no formalization of the quality index: e.g. Grossman and Helpman (1991a) – defining quality helps in characterising the theoretical difference between the vertical and the horizontal aspect of commodity differentiation.

When qualitative differentiation is introduced by assigning each commodity a value in a quality ladder, a strict separation between quantitative and qualitative aspects is required. For this reason, a commodity is referred to as a unit object, and its components as proportions of this object. Following the Lancastrian tradition, a commodity is outlined by its underlying *characteristics*, and is so depicted by the *allocations* of characteristics it contains.[10] In keeping quantitative and qualitative aspect of consumption separated, however, I depart from Lancaster's theory in assuming that the characteristic allocations lie in a unit interval, and that a commodity is properly identified only if the (proportional) allocations of the different characteristics it contains sum up to one.

[10] For a review of the theory of characterisitcs, see e.g. Lancaster (book).

Formally, I assume we observe a *characteristic set* $\mathbb{J} \subset \mathbb{N} : j \leq J$, where j stands for a generic characteristic and J is the number of elements in $\mathbb{J}$. Each j identifies one dimension in the *characteristic allocation space* $\mathbb{Y} \subset \mathbb{R}_+^J : \{y_j \in [0,1], \forall j \in \mathbb{J}\}$, where y_j measures the jth characteristic allocation. A commodity is defined as a $(J \times 1)$ vector $\mathbf{g} \equiv [y_1, y_2, ..., y_J]'$ in $\mathbb{Y}$, such that $\mathbf{g}'\mathbf{I} = 1$, where $\mathbf{I}$ is the unit $(J \times 1)$ vector. The resulting commodity set, denoted by $\mathbb{S} \subset \mathbb{Y}$, can be geometrically represented by a $(J-1)$ dimensional *simplex*, an object often used in studies involving probabilities. In figure 1, the simplex is portrayed by the gray wired triangle for a $J = 3$ case. Each point in the simplex represents a commodity (in the figure, one example is given by point $\mathbf{g}$), qualitatively different from all others.

FIGURE 2 – One Commodity in the Characteristics Simplex



The commodity set so obtained can be sorted by defining a set of proper *datum* points. Given the nature of the problem, a sensible benchmark is provided by human *needs*. These are assumed to define predetermined and objectively identifiable *ideal* objects, whose set is denoted by $\mathbb{V} \subset \mathbb{R} : v \in [0,1]$, where v indexes needs. I assume each ideal object finds concrete expression in a commodity equivalent (hereafter called *bliss*) in $\mathbb{S}$, denoted by $\mathbf{g}_v$. By comparing a generic $\mathbf{g}$ commodity to the bliss, I obtain a need-specific measure of qualitative differentiation. I name this measure *quality*, and denote it by $q(\mathbf{g}, \mathbf{g}_v) \in \mathbb{R}_+$ for some commodity $\mathbf{g}$ with reference to need v.

In this framework, it seems natural to imagine an inverse relation of quality to the Euclidean distance between the reference commodity and the bliss, denoted by $d(\mathbf{g}, \mathbf{g}_v) \equiv |\mathbf{g} - \mathbf{g}_v|$. Quality can thus be formally represented by the function $q(\mathbf{g}, \mathbf{g}_v) = f[d(\mathbf{g}, \mathbf{g}_v)]$, with $f'(\cdot) < 0$. Function f can be given a number of possible formalizations. A convenient way to define it is to allow quality to range in all positive real numbers as stated above. As shown below, other intervals may, however, be more appropriate in some contexts.

According to the framework developed so far, each commodity is able to satisfy, at least to some extent, all human needs. Although it possibly reflects reality in greater details, representing preferences in such a setting is extremely complex. The analysis is thus eased by assuming that one commodity can only satisfy one need. This univocal relation greatly simplifies preference representation, and the resulting demand functions take a more convenient form. A "qualitative priority" condition seems a sensible way to link each good to a single need: that is, a commodity satisfies the need for which that commodity is associated to the highest level of quality.

Denoting by $\mathbb{S}_v \subseteq \mathbb{S}$ the subset of commodities satisfying the vth need, I formally express the above condition as $\mathbf{g} \in \mathbb{S}_v \Leftrightarrow q(\mathbf{g}, \mathbf{g}_v) = \max_{z \in \mathbb{V}} \{q(\mathbf{g}, \mathbf{g}_z)\}$. If quality of a commodity is the same with regard to two or more needs, then it is assumed that commodity is in the subset with the lowest v. Under these assumptions, the subsets of commodities satisfying different needs are disjoint, i.e. $\mathbb{S}_v \cap \mathbb{S}_z = \emptyset$, $\forall v, z \in \mathbb{V}$. I name these subsets *varieties*. The definition of quality also provides a criterion for ordering the elements in each variety set $\mathbb{S}_v$. The quality space is denoted by $\mathbb{Q} \subseteq \mathbb{R}_+ : q \equiv q(\mathbf{g}, \mathbf{g}_v)$, where q is the level of quality associated to commodity $\mathbf{g}$. Using the definitions of quality and variety, hereafter each commodity $\mathbf{g} \in \mathbb{S}_v$ is identified by the pair $(v, q) \in \mathbb{V} \times \mathbb{Q}$.[11]

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[11] Notice that several commodities in the same variety may provide the same level of quality. Among them, however, only one will be actively produced. Since individuals will be completely indifferent in puchasing any of such commodities, demand will be set according to minimum-price criteria.

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